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ABSTRACT

The aim of this guide is to aid elementary teachers in individualizing instruction. Eight general objectives of teaching mathematics are listed, 11 topics are identified as being the scope of elementary mathematics, a one-page mathematics curriculum flow chart for grades K-8 is provided, student and teacher needs and evaluation are discussed in general terms, and three examples of individualized instruction are given. A section on mathematics laboratories includes directions for four activities along with a short list of math lab materials and books. A bibliography of 32 references on mathematics education is given. (DT)

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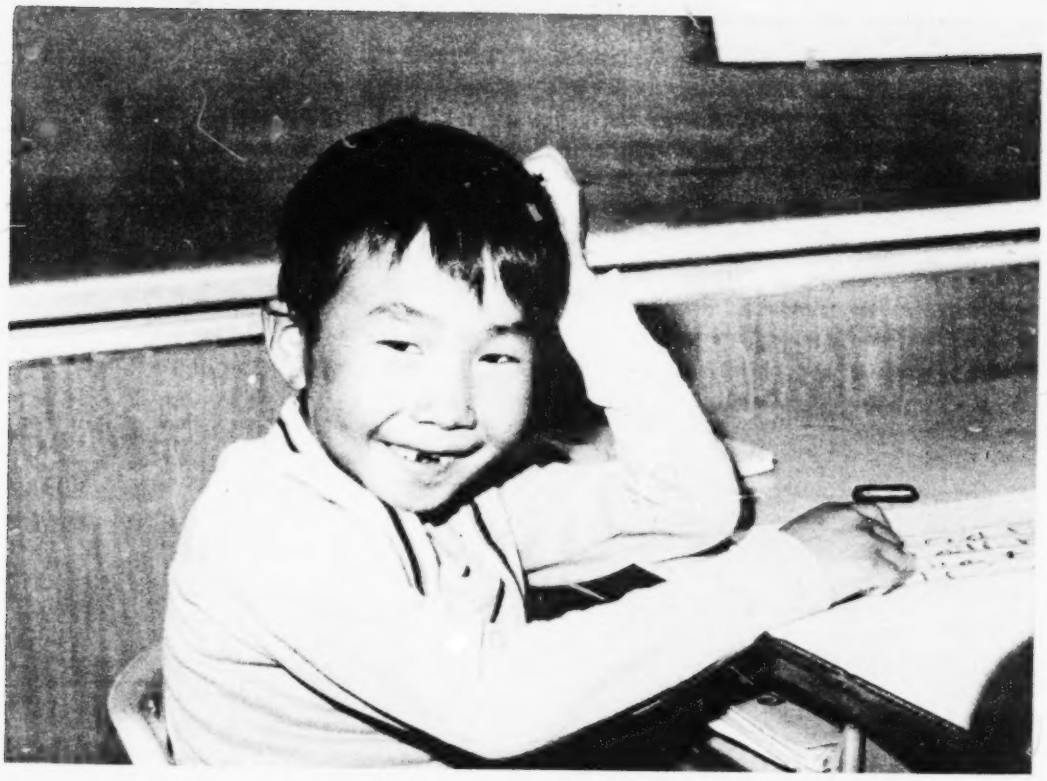
Elementary Mathematics

SE 015 962

ALASKA DEPARTMENT of EDUCATION, DIVISION of INSTRUCTIONAL SERVICES, August, 1971

ELEMENTARY MATHEMATICS

A Handbook For Teachers



I'M AN INDIVIDUAL!

Dr. Marshall Lind
Commissioner of Education

Dr. W. Russell Jones
Director of Instructional Services

1971

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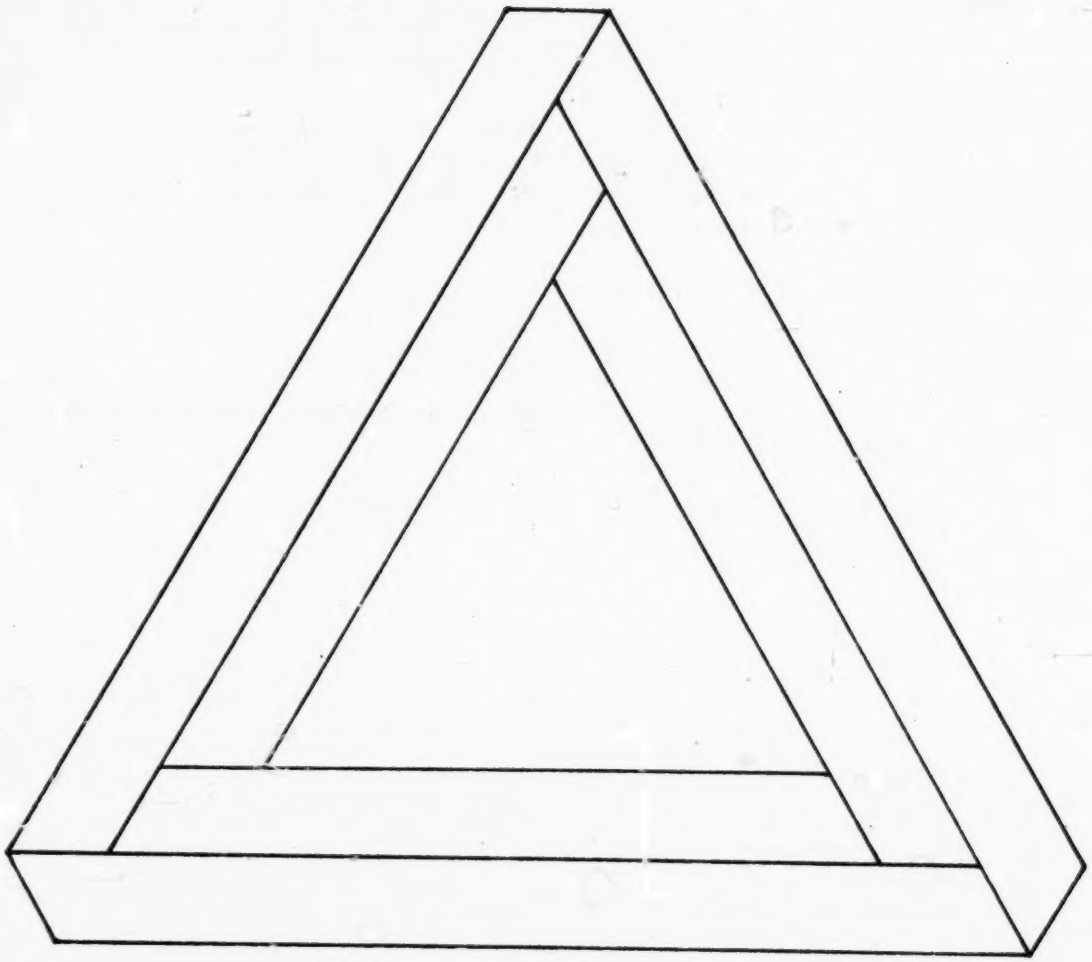


Philosophy

Education, to be living and effective, must be directed toward familiarizing children with those ideas which will enable them to function in the current world and develop in them the capacities which will prepare them for the future. This involves skills and processes of thought that are important socially as well as mathematically. These skills must be developed fully and to the maximum extent commensurate with a child's ability. This development must take place in each child, not just in a majority of the children.

This curriculum guide is prepared to aid teachers in accomplishing the goal of individualization. An individualized mathematics program will be achieved through an evolutionary process. Some of the principles that guided the committee toward the individualized learning concepts are as follows:

1. The child is the main concern of the teacher.

2. The child becomes a partner in the teaching-learning process.

3. Each child's positive self-image is enhanced by his successful experience in an individualized learning situation.

4. A child can perceive the beauty of the structure of mathematics.

It is our hope that this guide will be useful to individual teachers and to local committees. Even though some sections of this guide include specific examples, we realize that individualized instruction will take on a somewhat different nature from one classroom to the next.

Perceiving the BEAUTY!

General Objectives of Teaching Mathematics

1. To develop in the child the responsibility to plan his program of instruction, to evaluate it, and to acquire the drive to accomplish these goals.

2. To recognize and cultivate the inherent potential and creativity in each child.

3. To provide a firm foundation in the basic mathematic skills and principles.

4. To make mathmetics relevant to the individual and useful in his daily encounters with his society.

5. To develop a logical, sequential method of reasoning.

6. To enable a child to perceive the beauty of the structure of mathematics.

7. To enable a child to enjoy recreational mathematics.

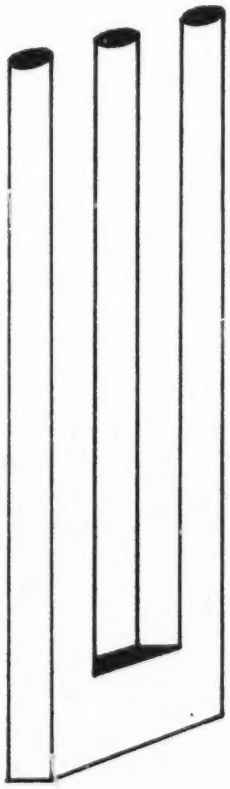
8. To foster in the child an understanding and use of machines in computation.

Scope

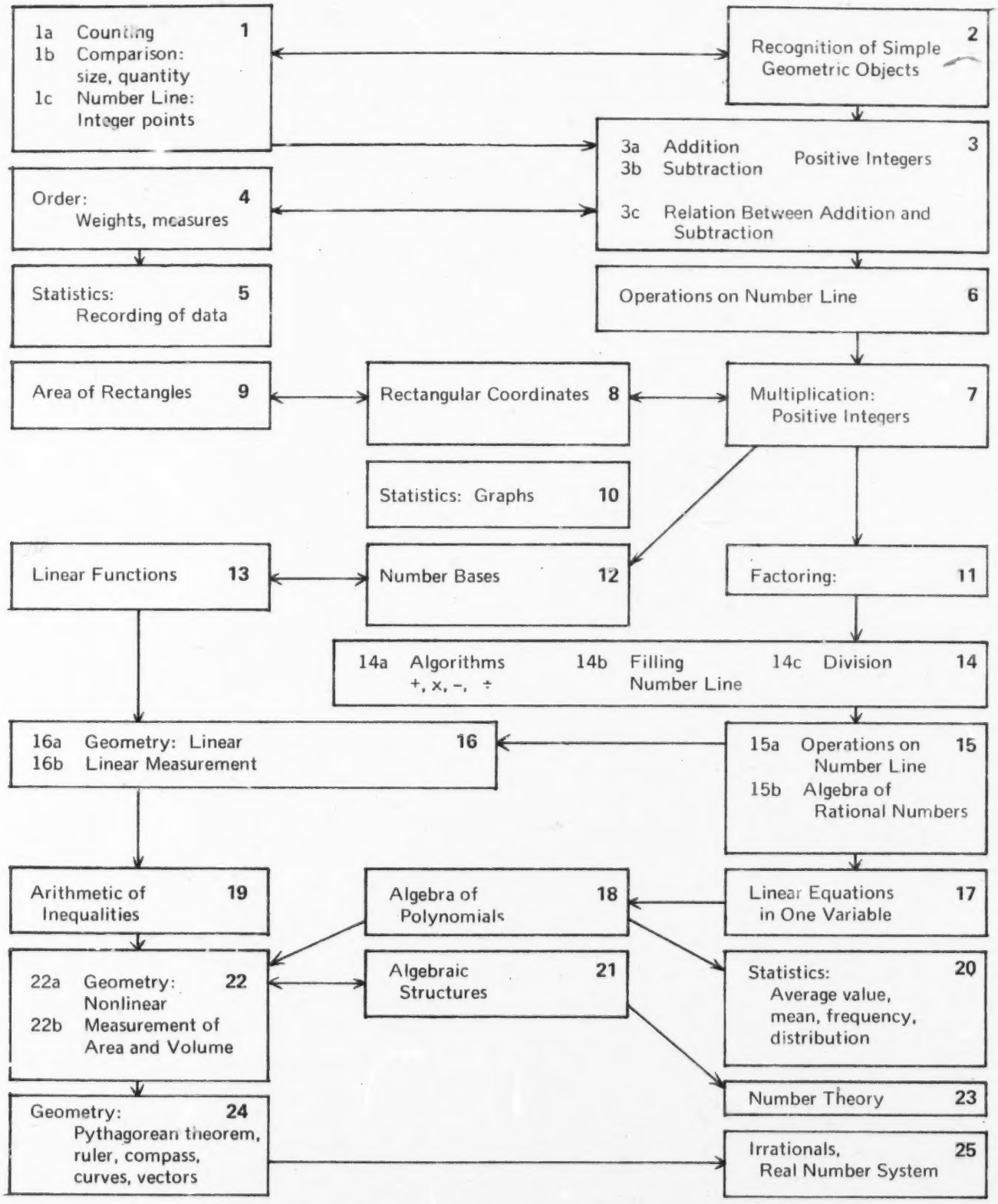
Mathematics in the elementary school is concerned with various aspects of the structure of mathematics. The teaching-learning sequence in mathematics can be likened to the piecing together of a series of jig-saw puzzles. One may begin with any piece, but each puzzle shows a picture of the entire structure. The earlier learner is given large pieces simple in outline and color. The more advanced learner is given smaller pieces more irregular in outline and richer in color.

The scope of elementary mathematics includes the following topics: (These topics are NOT in a sequential order nor necessarily disjoint.) See appendix for detailed scope.

Numbers and Operations
Special Topics
Functions and Graphs
Geometry
Sets
Number sentences and Solutions
Measurement
Statistics and Probability
Problem Solving
Application of Mathematics
Logical Thinking



MATH CURRICULUM K–8
Flow Chart



FLOW CHART COURTESY OF HOUGHTON MIFFLIN COMPANY

Implementation

Individual differences dictate individualized learning. The committee feels that this is the basis for all educational experiences. Individualization may cause a change in the educational philosophy of an individual teacher or, possibly, of a whole school system.

Individualized instruction must meet the needs of both the child and the teacher. Some of the child's needs are:

1. To feel that the subject matter is relevant to him.
2. To have immediate verification of his work.
3. To see progress in his program.
4. To accept the responsibility for learning.
5. To have open lines of communication with the teacher.
6. To gain recognition among his peers.

Some of the teacher's needs are:

1. To realize that a new, complete physical outlay is not necessary. Individualization can be accomplished with the existing books of varying difficulty.
2. To have the idea of individualized instruction understood by the children.
3. To recognize that the teacher and each child may have to move into the program gradually, developing individual responsibilities, as they work together toward individuality in learning.
4. To have open lines of communication.
5. To determine the child's level.
6. To develop a method of evaluating the child's progress. (The committee feels that this is best reported by a progress report rather than a standard report card.)
7. To improve his subject matter competence to meet the needs of every individual.
8. To realize that the teacher is now on a one-to-one correspondence with the pupil, even though he may be working with small groups or total class sections. Individualization of instruction does not mean isolation. An opportunity for group action and reaction is necessary.

Evaluation

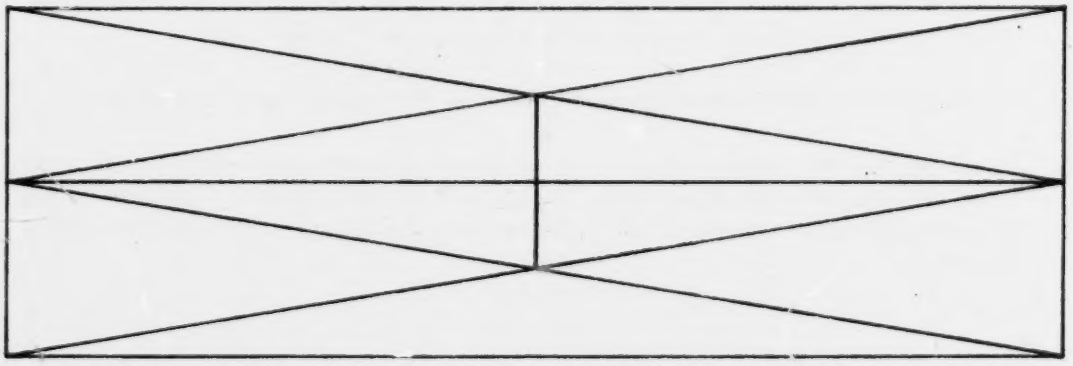
The changing role of the teacher is most evident in the approach to the evaluating process. The time previously spent in lecturing in a traditional classroom situation will now be spent in managing student progress and in stimulating the flow of ideas. Some factors involved in evaluation are:

1. Informal testing—a few teacher-made problems covering basic concepts.
2. Teacher observation—does the student work confidently and accurately?
3. Teacher-child conferences—conversation involving reasoning, use of measurement, estimating, etc.
4. Formal diagnostic testing.
5. Since children are at differing stages of development, we must evaluate the mathematical experience of each child to determine his starting level.

A continuous appraisal of the child's progess is necessary to help him move from concept to concept and from one level to another. The progress record kept by the child and the teacher is an excellent tool to encourage the child's self-directed mathematical growth and to help him determine his own needs. This record is very useful in reporting the child's progress to his parents.

Evaluating the school curriculum is also part of the total evaluation. This will be a continuing process of measurement and of judgment as we compare the school's curriculum to the current objectives that we wish to accomplish in the mathematics program. Even a first-grade child can answer questions such as: Do you like math? What part do you like best? What did you learn in math? What would you like to learn next? What part was most difficult?

Curriculum evaluation should involve parents, administrators, teachers and children. Teachers, parents and administrators will be able to assess the correlation between objectives and the curriculum, but only the child can supply the answers as to how meaningful the mathematics program has been to him.



How many triangles can you count?

Examples of Individualized Instruction

Before describing a typical math class, it should be emphasized that individualizing instruction is by nature very personal. Each teacher will find a unique way of individualizing depending on his personality and that of his class members.

The following models are examples of individualized classroom instruction which are effectively being used today in Alaskan schools.

Model A

This model evolved from a need to adapt teaching methods to a mobile student population; to give each child the maximum learning opportunities while he was in the group. The model has been used successfully from primary through junior high levels.

The teacher has informally evaluated the child and has devised a possible approach for this child at his level.

To begin the program a very simple concept, such as sets, is introduced to the entire class. Many will already be familiar with sets; informal discussion and demonstration by the children will occur. Several copies of basic texts, grade levels 1 — 8, will be needed to meet individual differences.

Assign each child material on the concept at his level or one level below. There will usually be two or three children working at the same level. As each solves his problem, he compares his work with others in his group, discussing methods, solutions, etc. He may need help from someone who has finished that level of instruction, thus he asks a friend to assist the group or to help him individually. As children work, frequent discussions occur with the problem illustrated on the overhead projector or the chalkboard. The entire class usually becomes involved in the instruction several times during the work period.

The teacher is the expediter, moving casually from group to group, never helping unless an individual or group cannot solve their problem independently.

With many levels of difficulty of the concept being discussed and illustrated, the children may see immediate application of the concept. The children working at lower levels may especially benefit from this discussion. A child may join another group and work through problems at several levels as he becomes more expert at using the concept to solve problems, or he may work alone, checking with others as he progresses. If a child discovers a really stimulating idea he should be encouraged to pursue it further even if it branches out from the original concept.

An individual progress report for that concept is stapled to a large folder. As the child successfully completes several problems and compares them with his contemporaries, he checks off his progress and deposits his paper in his folder. If he is not ready for the next

level, the group he has joined will either help him master difficult points or tell him he needs more practice before he is ready to move forward with them. (The teacher can "expedite" at this point.)

Each child works at his own rate. As he needs to know he asks, and is instructed individually by other children or the teacher in small group interaction and in discussions involving the entire class. Children may be working from a level of manipulating objects, performing simple operations with sets or using sets to solve problems with positive and negative integers. Each concept has such a broad range of application and leads so easily to the need for other concepts that the individual child controls his own sequential development in the program.

The teacher constantly assesses each child's performance as she moves around the room, knowing where the child is working and his rate of progress. Papers are available in each folder for re-checking. More evaluation occurs as the teacher asks each child to work one or two problems at his level and to discuss the operation with her. Formal testing at his level may be used at intervals if the teacher feels it is necessary.

More than one series of texts should be used so the children are familiar with several methods of presentation. This helps the children discover many methods of problem solving. Formulating and solving problems of their own is a most desirable learning situation.

Individualizing is a social process and cannot take place when each child is working strictly by himself. There must be free movement of students among groups to enable communication of differing ideas and approaches. The teacher must also move constantly, drawing out students, stimulating the flow of ideas and giving problem solving situations to encourage children to apply mathematical concepts to real-life situations. Frequent group discussions must take place to stimulate the entire class.

Self discipline develops as the child sees himself as an individual, learning in an individual way, yet involved in the process of assisting others to learn. Children are sharing the enjoyment of mastering skills and understanding concepts. They are learning to work in groups, developing a respect for the opinions of others, and practicing many other aspects of becoming effective citizens.

Model B

Model B is a type of individualized instruction that a fifth-grade teacher in Alaska has been using for several years. She has developed a method that works well for her. It is our hope that many of her ideas will be useful to you in your classroom.

The class starts when the student secretary returns the daily work and quizzes. Each student has already checked and corrected with green pencil his daily work before turning it in. It is reassuring to know the teacher has glanced over the paper and can quickly correct any serious error before it becomes a set habit. As soon as a student has filed yesterday's math work, he can glance at his individual progress sheet on the front of his folder (made

previously by the teacher) to see where he should begin today.

It is not uncommon for the math ability range in the class to be three or four grade levels. The teacher avoids pre-judging a child's potential which may have been latent under a conventional teaching system. Although she has no more than an average mathematical background, she has confidence in her own ability to help the child determine his starting point. She finds individualizing her math class is easier since acquiring many supplementary math books and aids to fit this wide ability range.

The atmosphere in this classroom is one of student freedom to talk and move around the room. One of the reasons the teacher is so apostolic toward this method of teaching is because her discipline problems with the children have nearly disappeared. In a setting where every student is challenged to work at his own individual rate and level, each student is able to experience success. This success builds positive attitudes in the students, which may carry over to their offspring in a few short years.

She believes many small discussion groups and some full class discussions are an essential part of any method of individualizing instruction. She insists that understanding of math concepts is an important aspect of learning which some children may not gain unless these concepts are developed and repeated in small and large discussion groups. Some children learn much more quickly from hearing another student's explanation than from the teacher's explanation. The teacher is also aware that she must always take time to listen to each individual.

There is no set schedule of individual study time, small group study time or large group discussion time. She feels one of the real rewards of individualization is the openness with which her students discuss their particular progress, insights and difficulties with classmates and with her. This is in contrast to the conventional situation where a teacher may feel hopelessly frustrated because a student is completely unresponsive.

An outstanding feature of this instruction is the eager attitude each child shows toward his mathematics. Some students may not cover too much ground during a given period of time. However, with this feeling of eagerness, learning is taking place.

Students want to be personally involved in what they are doing, and this method of individualizing mathematics is an excellent way for each student to discover material that is uniquely suited to him as an individual.

Model C

The notion of individualized learning pervades the instructional method one teacher has used over the past ten years in mathematics and science at the traditional seventh, eighth and ninth grade levels.

This narrative describes more clearly the conduct of a class at the ninth grade level. The implementation may vary considerably depending upon the maturity of the individuals

involved and make-up of the class, but the general notions and ground rules apply at all levels.

Every response to a given classroom stimulus should be governed by a consideration of these questions:

1. Is it the best line of action for the student?
2. Will it contribute to the making of a responsible citizen?
3. Will it interfere with the learning process of other people?
   (We do not have the right to interfere with another's learning process.)
4. Will it result in an unnecessary contest between teacher and student?

With every student there must be a continuous program of self-evaluation and goal setting. With older students this can be quite objective and straightforward (i.e., why are you in this class? what would you like to go back and re-do? etc.) but with younger children this procedure might become more exploratory, with a gradual sifting and sorting, before realistic goals are established.

Generally, the greater the supply of resource material the better, but these classes have functioned well with just one textbook series until other materials were accumulated.

Time must be spent throughout the year to reinforce the notion that each person can and will learn. When the youngster accepts this notion and a starting point with a realistic goal has been established, the teacher can then relax his grip and let the student proceed along his learning path—working, assisting, and receiving from activities with his classmates.

This working-helping phase might well be a very quiet period; but then, again, it could be noisy. Caution: Do not be hasty to judge, as degenerative, activity among a group of students. Very often a burst of laughter accompanied by back-slapping can indicate that someone in the group "has finally seen through." A group of students may seek permission to go elsewhere (they specify where) to study together when they feel the resulting activity will lead to a disruption of the class as a whole, e.g., when they wish to view single concept filmstrips or movies, or to practice fundamentals.

What about letting the student use his study time to prepare for a test in another subject for next period? By all means! When the young person is functionally mature enough to be open with you, you have a lot going for you; and at that stage of development, discipline problems begin to disappear!

The conventional notion of grades needs to be modified so that each child is graded on his own progress (along several avenues to be discussed later) and that work is the sole requirement for an "average" grade. Expect spurts in performance as he proceeds along his learning path.

Beyond this "average" grade, a subjective judgment of "attitude toward the class" is woven into the fabric of the evaluation. (Note: This takes into consideration such things as:

absenteeism, late arrivals, working relation with classmates, willingness to explore and assist. It DOES NOT INCLUDE attitude of student toward the teacher. Teaching is not a popularity contest. The teacher should expect and encourage differences when opinions are aired in an open atmosphere. Differences of opinion should not be interpreted as an insult by an individual.)

High in priority on the list of student needs is immediate verification of his work.

The need is fulfilled in three ways:

1. Consensual validation: pupil to pupil. Typical is, "I got this. Wha-ja-git?"

2. Access to the teachers' edition textbook.
   A ratio of one book to seven students seems adequate. (Notes to the teacher in the textbook are very helpful to the child.)

   a. For use in school during the day.

   b. To take home nights and weekends. Post a definite schedule when each child can have a book on one night of each week. He can then organize his study time in other subjects around this.

3. Access to the teacher. Although it may not be possible in all situations, the teacher should be available to assist any time during any reasonable hour. This helps to convey the idea that the teacher is concerned with and interested in the progress of the child.

The need for each child to see progress.

1. Must have access to the record book (no longer called the "grade" book.)

   a. To check his own progress—or to determine a new starting point.

   b. To assist in recording his own work.
      Question: Can you trust the child to record accurately? Yes, especially when learning rather than teaching is the goal. A prevailing attitude of, "Who are you really kidding!" becomes quite commonplace.

2. Student Work Sheets (tests) at the option of the student. Single concept work sheets encompassing a depth in understanding are always available. Students take these when they feel they are ready.

3. Teacher Evaluation (tests) at the option of the teacher. A cumulative work sheet covering the work done by any and all of the students may come at any time. All children may not finish this work sheet (test) in the time allowed. Caution: It is difficult to use a work sheet such as this and not discourage some children. An attitude of "I'll do all I can" must prevail. The evaluation also includes random checks of papers and observations made during the study-work time.

Note: All these evaluations (tests) are considered to be learning devices and are treated as such. When learning is stressed, there seems to be a tendency to study to learn rather than study to pass tests and, at that time, it becomes unnecessary to guard against cheating.

There is a need for a sounding board or "feedback" path by both teacher and child. (Understand "feedback" to carry this meaning: a means to improve the fidelity of the system.) This implies a DYNAMIC SYSTEM IN CONSTANT CHANGE!

Student evaluation forms carry three avenues of questioning on both subject matter and citizenship. These avenues are provided to:

1. Explore the child's progress:
    a. As he expects himself to be moving along.
    b. As he would like to be doing.
    c. As his school and community expects him to be doing.
2. Give an opportunity for re-establishing goals.
3. Give an opportunity for comment and constructive criticism.

The student's role becomes one of having control over his own progress. He can explore, probe and experiment both horizontally and vertically over a wide area. Caution: Do not be disturbed if a child goes back to ro-do work done at an earlier time because, perhaps now, he finds a need for greater understanding.

A vital role in this entire process is the teacher's recognition of himself as a professional person who keeps each child somewhere within his own target area. The teacher assists when needed and redirects when the child has gone too far afield.

The teacher should make an effort not to interfere in a learning process of a child. Time is required for these processes to take place. The teacher should keep his hands out of the learning experiment until he is asked to assist or until he sees the child headed for a blind alley from which the child cannot extricate himself.

The teacher may be compared to a flywheel, absorbing the excess energy and then feeding it back as the needs of each individual child dictate.

The teacher is called upon to render clinical judgment in many simultaneous situations. Mistakes will be made. Errors in judgment become immediately recognizable and remedial action is possible while the learning process continues.

Priority should be given to preparing substitute teachers. On one occasion, the children provided the key to satisfying the requirements of a reluctant substitute who demanded a

definite set of lesson plans before he would accept the job. One boy volunteered, "Each of us can just write down what we plan to be doing for the next ten days and give him that." The teacher handed his substitute a sheaf of papers (one for each child) for each class, and departed. The substitute dutifully checked each sheet on each child for the first two or three days and then, happily, found he could spend his time more productively assisting the children in their learning activities.

The question of discipline always comes up. A punishable offense is one in which someone interferes in the learning process of another. This is a relatively rare offense. When there is some doubt as to what a burst of activity might mean, the question, "Are you helping?" or "What are you supposed to be doing?" solves a lot of problems. When a child is restless and can't seem to get on with the tasks at hand, he can always straighten books on the back shelf. Generally, the discipline problems that do occur are minor compared to those of the old lock-step classroom.

How does the teacher place a child at his best working level and how does he make sure that the child moves along at a satisfying rate?

Several devices can be used to achieve this purpose. They include:

1. Standardized test results from the previous spring.

2. Observation of a chalkboard game played with simple, fundamental arithmetic concepts. This game is much like the old-fashioned spelling bee and is held once a week, with the losers supplying ice cream to the winners at the end of each grading period.

3. Individual and group conferences with the child in which he assesses himself.

4. Current evaluations and worksheets.

If the child is new to the school, refer to the evaluation given by his previous teacher. However, the teacher may prefer to observe the student for himself because a child changes from day to day. He should be a better-organized individual today than he was yesterday. If this does not appear to be the case, then, the teacher had better find out why.

ASSESSMENT OF THE CHILD SHOULD BE CONTINUOUS.

In summary:

1. The conventional notion of grading is not compatible with an individualized learning situation.

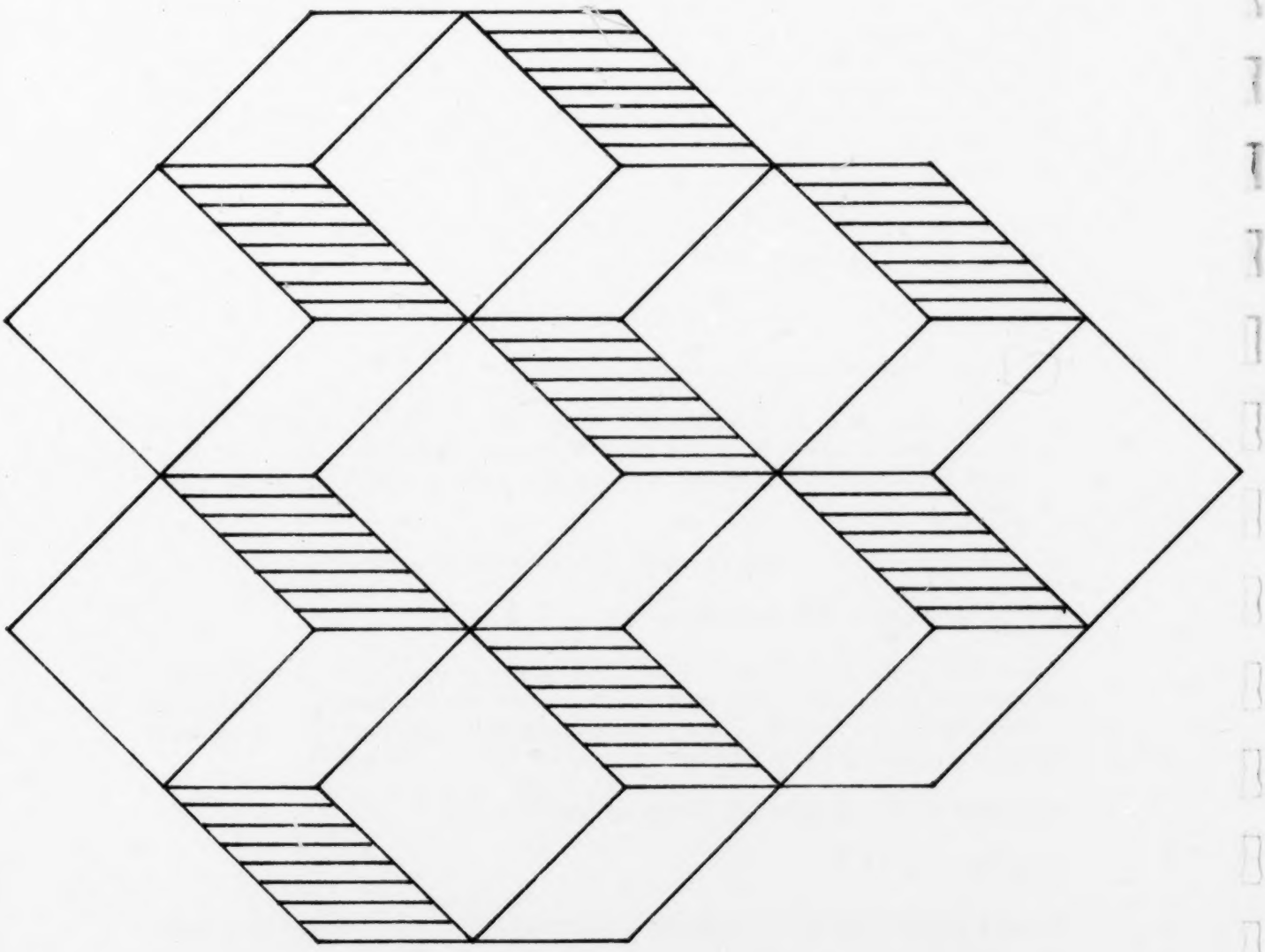
2. The child must have control over his learning progress.

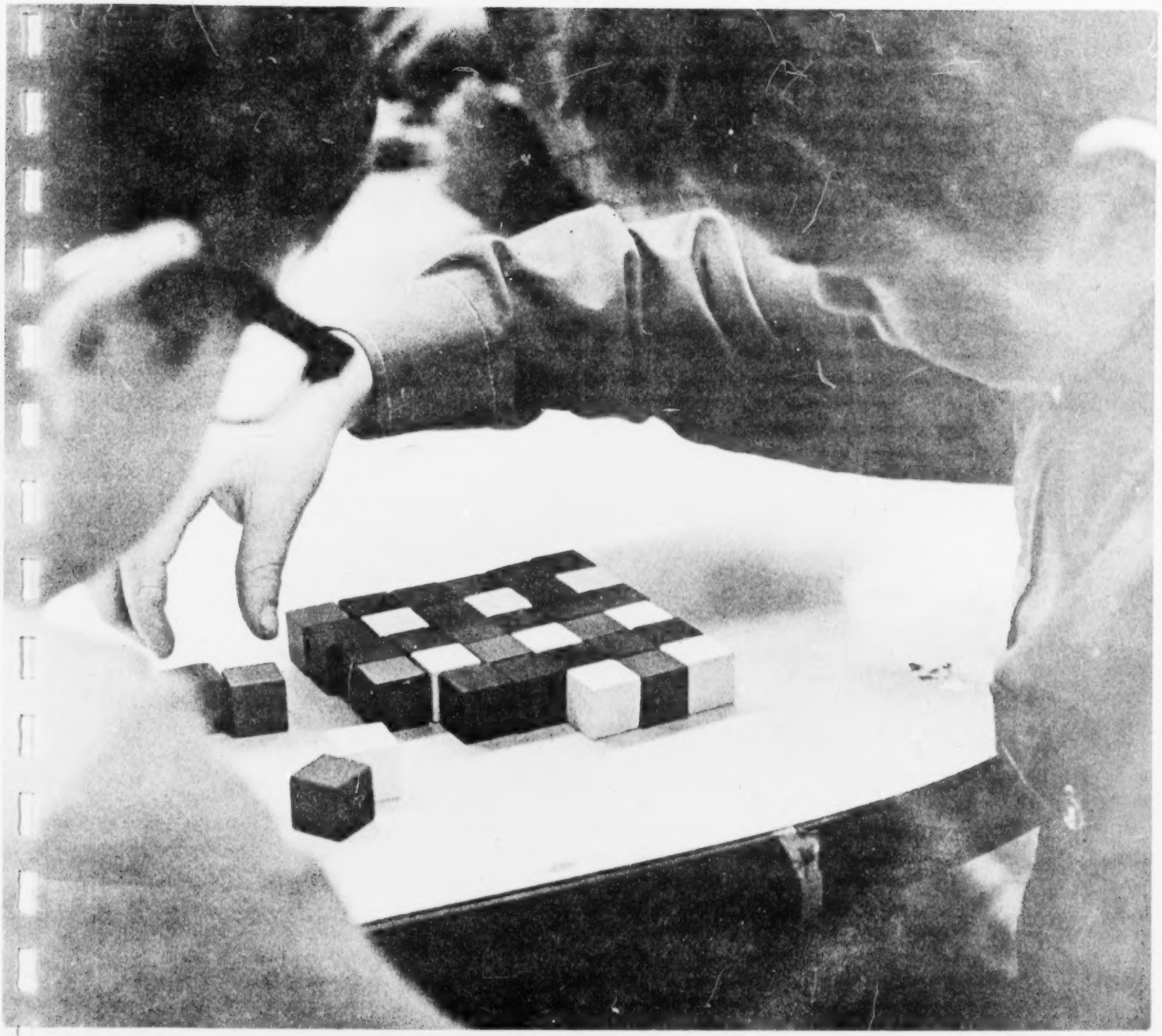
3. Each child should understand that he is expected to do his best and that he will be judged on his progress.

4. There can be NO learning contest between the teacher and the child.

5. It is vital that the teacher recognize his role as a professional person who keeps EACH child somewhere within his own target area.

6. Teaching can be satisfying and rewarding WHEN THE EMPHASIS IS PLACED ON LEARNING.





Mathematics Laboratory

The laboratory approach to teaching can play an important role in the child's understanding of mathematics. The materials may be pupil constructed, teacher constructed, or commercial.

The materials should be readily available so that the student may use them at any time. This may be in a corner of the classroom, or possibly in the media room.

The laboratory learning progresses from the concrete to the abstract concept(s) as the children work with the materials.

Some examples of laboratory situations follow:

Areas of closed geometric figures

Materials: 1" graph paper, scissors.

Objectives: The student will discover the meaning of area, develop a method of finding the area, and evolve related formulas.

Procedure: The student will cut 1" squares from the graph paper, and use them to construct a square. By counting the number of squares he will find the area. Using the same method, he will proceed to the area of other figures, rectangles, triangles, and irregular forms.

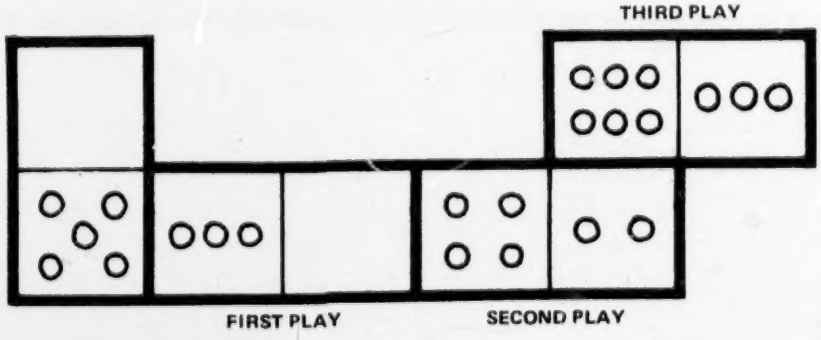
There will be countless solutions by the students. As a group they will then compare, and evolve suitable conclusions.

Dominos

Materials: Double nine dominos.

Objective: To gain proficiency in the use of multiplication facts.

Procedure: The students play the game by placing the dominos end to end so the total of the two symbols equals a multiple of a given number. In the following illustration, the answers are multiples of 4.

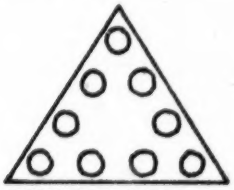


Magic Triangle

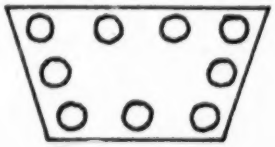
Materials: Posterboard and colored paper discs which are consecutively numbered.

Objectives: To gain skill in addition facts.

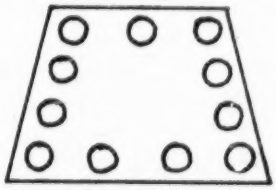
Procedure: Give the students a magic triangle. Then encourage them to make up their own magic figures. Below are examples from a fifth grade class.



EACH SIDE TOTALS 17



EACH SIDE TOTALS 16



EACH SIDE TOTALS 22

Survival

Materials: Catalog and newspapers.

Objectives: Computational skills and reasoning.

Procedure: There is an uninhabited island at ___ Latitude. It has no man-made facilities. With 3 or 4 friends, plan what supplies and equipment you will need to survive on the island for the month of July.

The only source for ordering equipment and supplies is the mail order catalog and grocery ads from the newspaper.

You have unlimited money, but can only ship 1,000 pounds of materials to the island. List by name of item, shipping weight in pounds and ounces, and the price.

Be able to justify your use of each item. If another group successfully challenges its usefulness, you must drop that item and as a penalty drop one other item of your choice. (Vary by changing time limit, latitude, season, or by imposing a monetary limit, etc.)

Freedom to Learn by Edith Biggs, published by Addison-Wesley, contains valuable information and practical examples for teachers planning to implement a math laboratory.

See Resources for Learning section for a partial list of math laboratory materials. Page 19.

Resources for Learning

A rich, stimulating environment facilitates learning. This encompasses many types of resources: e.g., human resources, firsthand observation and experience, exploratory and experimental material, opportunities and materials for self-expression, printed materials, audio-visual materials and newer learning media.

Human resources — including the children, people from the community, parents, other teachers, . . .

Firsthand observations and experiences — has the learner had arithmetic experiences outside of the classroom that he can share with the class? Is he a paper boy? Has he built things with his dad? Or does the teacher need to build these experiences? Can he collect the money and keep the records for the classroom book club? Can the class visit the grocery store? the bank? etc.

Opportunities and materials for self-expression — verbalization through discussion, role playing, students tutoring other students, student-made tests, problems and systems created by students, student-built models, charts, maps and scale drawings, self-established goals, . . .

Printed materials — textbooks and supplementary texts at varying levels, books on the history of mathematics, readiness booklets, enrichment booklets, newspapers, self-evaluation materials and meaningful practice materials designed for use by the child without direction by the teacher, exercises to give math practical applications to the child's daily life, use of measurement, objects, optical illusions, geometric designs, open-ended sequences and patterns, puzzles and conundrums.

Audio-visual materials — blocks, flannel board, counter board, games, flash cards, abacus, rocks or marbles to help visualize numbers, clocks, toy money, charts, individualized computational skills kits, programmed learning kits, overhead projectors, tapes, films, filmstrips, records, computers, television, commercial games. (See bibliography printed in November, 1968, *Arithmetic Teacher.*) Some of the best aids are teacher made.

Math Laboratory Materials

Inexpensive Lab Items

Developmental Math Cards, Sets A-L @ 3.96/set — Addison-Wesley Co.

Mathematics Laboratory — $15.00 — published by Book — Lab, Inc., 1449 37th St., Brooklyn, New York 11218 Kit No. 3044

Multiplying Machine — published by Selective Educational Equipment, Inc., 3 Bridge St., Newton, Massachusetts

Tri-Ominos — Pressman — available Northern Commercial Co., Anchorage, Alaska

Quibie — Parker Bros. — 3D Tic Tac Toe — available Roberts, Mt. View, Alaska (or try your local toy store and hobby shop.)

The Winning Touch
Sum Fun
Smarty

Educational fun games—available Alaska School Supply, Anchorage, Alaska

Cross Number Puzzles — Ideal
Number Rummy - Kenworthy Ed. Services
Make One — Dolch
Kaleidoscope Puzzles — Ideal

Available from Alaska School Supply

Cribbage — Thrifty Drug Store, Mt. View, Alaska
Dominos — Thrifty Drug Store, Mt. View, Alaska
Numble — Cross Number Game, Roberts, Mt. View, Alaska

(Or try your local toy store and hobby shop)

Books:

Fun with Puzzles — Joseph Leeming
Zero to Zillions — Irwin Weiss (Scholastic Book Services, 904 Sylvan Ave., Englewood Cliffs, New Jersey 07623)
Arithmetic Games & Activities — Wagnee, Hasier & Gillaby
Mathematics Games for All Grades — Lola J. May

Reprints of: *An Annotated Bibliography of Suggested Manipulative Devices* for the lab may be obtained by writing to PERCY, 650 International Airport Rd., Anchorage, Alaska 99503.

APPENDIX

Topics of the Scope expanded:

Number and Operations

Place Value
Notation
Natural Numbers
Whole Numbers
Integers
Rational Numbers
Real Numbers
Bases Other Than 10
Basic Principles
Algorithms
Number Line
Cardinal and Ordinal
Number Theory — Primes, Even and Odd, Etc.
Pattern Recognition
Computational Short Cuts
Multiple Operations
Modular Arithmetic

Measurement

Recognition of Shapes and Forms
Building a Conceptual Framework of the Notion of Measurement
Concepts of Length, Area, Volume, Weight
Estimating and Rounding Off
Systems of Measuring Units
Money
Time
Temperature
Concepts of Liquid — Cups, Pints, Quarts, Gallons
Ratio and Proportion

Statistics and Probability

Note: By introducing Statistics and Probability theory early, we are giving the student a chance for a higher level of sophistication with which to interpret what he sees in ordinary life.

Sets
Sample Spaces and Events
Compound Events
Statistical Probability
Mathematical Expectation
Conditional Probability

Functions and Graphs

Note: A function is a relation between two sets.

Ordered Pairs
Graphing Inequalities
Graphing of Solution Sets
Axes
Coordinate Geometry
Number Line
Bar and Circle Graphs
Pictographs
Maps and Charts
Scale Drawing
Line Graphs

Problem Solving

Note: Logical problem solving is a primary function of elementary mathematics.

Graphs, Scale Drawings, Diagrams
Estimation
Average
Nonwritten Solution Sets
Application to Real-Life Situations
Ratio and Proportion
Translation from Number Sentences to Short Stories and Vice Versa
Multiple Operations
Number Line
Applications to Other Disciplines

Geometry (Informal, Nonmetric)

Recognition of Shapes and Forms
Number Line
Points, Lines, Planes, Simple Closed Curves
Solids
Coordinate Geometry
Congruence
Pythagorean Theorem
Parallel, Perpendicular, Transverse
Spatial Relationships
Constructions
Symmetry
Formal Attention to Abstract Concepts
Topology

Applications of Mathematics

Vocational
Commercial
Recreational
Personal
Physical Environment
Other Areas of the Curriculum

Sets

Element
Equivalent Sets
Non-Equivalent Sets
Cardinal Number
Subset
One-to-One Matching
One-to-Many Matching
Number Pair
Empty Set
Solution Set
Union
Intersection
Universal Set
Venn (Euler) Diagram
Sets in Geometry
Probability and Statistics
Cartesian Product
Disjoint Sets
Partitioning

Logical Thinking

Truth Tables
Axioms and Theorems
Formal and Informal Proofs
Quantifiers (all and some)
Negation
Conjunction (and)
Disjunction (or)
Conditional and Bi-Conditional Statements (If...then...)
Converse, Inverse, and Contrapositive
Inductive and Deductive Reasoning
Patterns and Sequences
Use of Logic in Developing New Concepts
Informal Logic in Computation and Problem Solving

Number Sentences and Solutions

Addition
Subtraction
Multiplication
Division
Notation
Decimals
Rationals
Variables
Constants
Open and Closed Sentences
Solution Set
Inequalities

Special Topics

History of Mathematics
Use of Computational Aids
Absolute Value
Determinants
Trigonometric Ratios
Paper Folding
Modular Arithmetic
Other Bases
Artificial Operations

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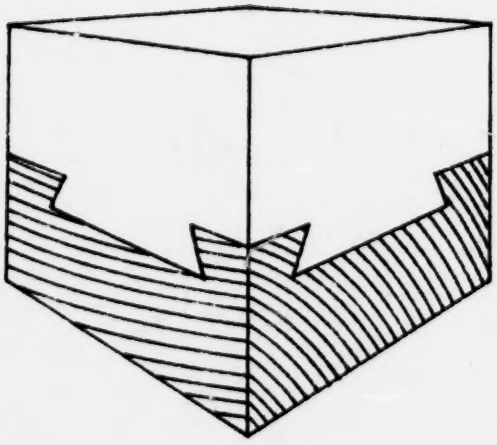
The two piece cube

Explain how this cube is constructed.
It may be taken apart and joined together without breaking.
The back faces look like those visible.



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ELEMENTARY MATHEMATICS

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Commissioner of Education

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